Prob. of events. eg $P(A)$

Prob. of complex events. eg. $P(A \cap B)$

Random Variable.

Categorical

X is the color of the student that enters the door next.

$\{X = Red\}$ $\{X = Blue\}$

Quantitative

Generic Distribution.

Special Distribution.

Generic Distribution.

X	0	1	2	3
$P(X=i)$	0.2	0.3	0.1	0.4

$$P(X \leq 1) = \frac{\text{Area}(\{X \leq 1\})}{\text{Area}(S)}$$

$$= \frac{0.2 + 0.3}{1} = 0.5$$

Similar to categorical variable & prob. of events.

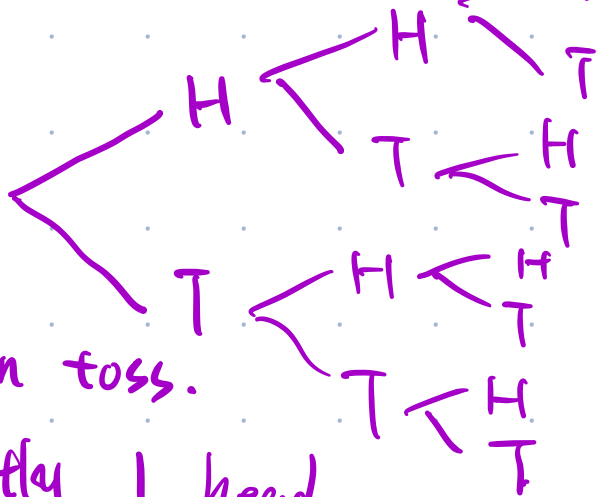
Special Distribution

- discrete 离散
  - Binomial distribution
  - Geometric distribution
- continuous 连续
  - normal
  - uniform

eg 1.

3 coin toss.

exactly 1 head.



HHH
HHT
HTH
HTT ✓
THH
THT ✓
TTH ✓
TTT

3

$$P(1 \text{ head}) = \frac{3}{8}$$

total # of outcomes

Binomial Settings (Remember!!!)

1) fixed number of trials n

2) only 2 outcomes each trial
   sucess or failure.
   define yourself

3). fixed sucess prob. p.

4). independent trials.

X represent the number of sucess in n trials

Binomial formula. ← Binomial PDF.

$$P(\boxed{X = k}) = \binom{n}{k} p^{k} (1-p)^{n-k}$$

PDF

eg 2. $P(X=2) = \binom{5}{2} 0.2^{2} (1-0.2)^{3}$

$$\binom{5}{2} = C_5^2 = \frac{5!}{2!\,(5-2)!}$$

$$\binom{n}{k} = C_n^k = \frac{n!}{k!\,(n-k)!}$$

Binomial CDF.

$$P(X \le k) = P(X=0) + P(X=1) + \cdots + P(X=k)$$

$$X \sim \text{Binom}(n, p)$$

R.V.

服从

二项分布.

max trials

prob. of success for each trial.

$$E(X) = \mu_X = np$$

expected value

mean of X

$$Std(X) = \sigma_X = \sqrt{np(1-p)}$$

$\uparrow$

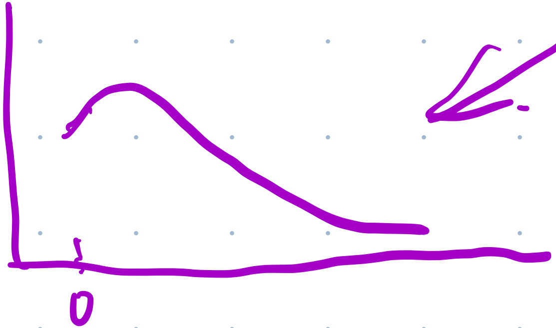
std. of X.

当 $p \to 0.5$. $X \rightsquigarrow$ symmetric

当 $np \geq 10$, $n(1-p) \geq 10$ 时. $X \to$ Binom $\to$ Normal

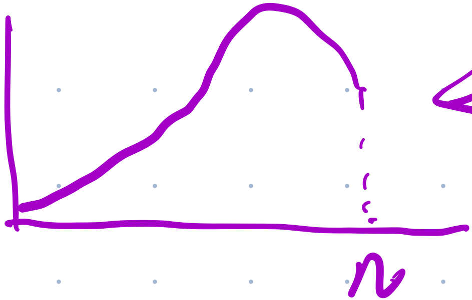
当 $p \to 0$ 时. 且 $np < 10$ 时. 左截断

skew to the righ.



当 $p \to 1$ 时，且 $n(1-p) < 10$ 时，右截断

skew to the left



$X \sim$ Geometric$^{\text{几何分布}}(p)$.

X 是多次实验，第一次成功的序数.

用计算器

$$\begin{cases} P(X=k)=(1-p)^{k-1}p \leftarrow \text{PDF} \\ P(X\leq k)=P(X=1)+P(X=2)\cdots+P(X=k) \end{cases}$$

first, until.

$E(X)=\frac{1}{p}$

Expected value

$Std(X)=\sqrt{\frac{1-p}{p^2}}$

only skew to right