

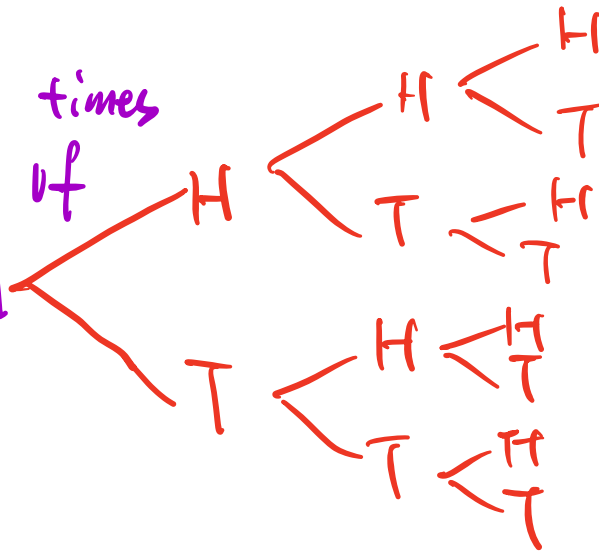
Example 1

toss coin 3 times

calc. prob. of

exactly head

1 time.



HHH

HHT

HTH

HTT

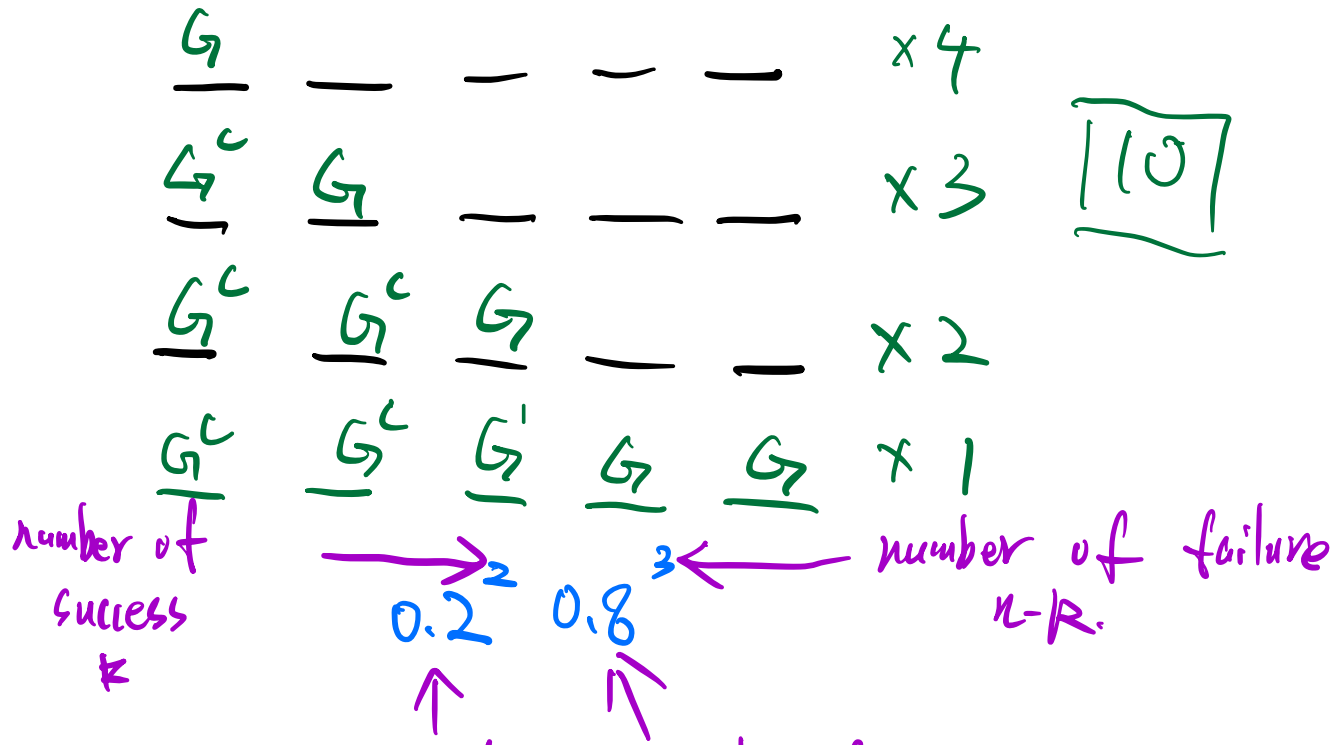
THH

THT

TTH

TTT.

Example 2. 2 Green 3 pink 5 blue balls
in the box. randomly pick 5 with
replacement. prob. of exactly 2 marbles.



prob. of
Success

p

prob. of failure

$(1-p)$

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$\uparrow$

$$= \binom{n}{k} p^k (1-p)^{n-k}$$
$$= \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k} \quad \text{当 } X=k \text{ 的时候}$$

Binomial Distr.
PDF $\leftarrow$ function
 $\uparrow$ Probability density
 $\uparrow$

$$P(X \leq k) \leftarrow$$

Binomial distr.
CDF $\leftarrow$ function.
 $\uparrow$ cumulative $\uparrow$ distribution

$$= P(X=0) + P(X=1) + \dots + P(X=k).$$

当 $X \sim \text{Binom}(n, p)$ 时.

$$\begin{array}{ccc} E(X) & = & \mu_x = n \cdot p \\ \uparrow & & \uparrow \\ \text{Expected Value} & & \text{Mean} \end{array}$$

$$\text{Std}(X) = \sigma_x = \sqrt{n p (1-p)}.$$

For Binomial $0 \leq X \leq n$

n, p 对 shape 的影响.

当 $np \geq 10$ 且 $n(1-p) \geq 10$ 时. binomial 近似于 normal distribution.

当 n 越大, p 不变时.

$E(X) = np$. $n \uparrow \Rightarrow E(X) \uparrow \Rightarrow$ 图形右移.

$Std(X) = \sqrt{np(1-p)}$ $n \uparrow \Rightarrow Std(X) \uparrow \Rightarrow$ std 变大.
图形变矮胖.

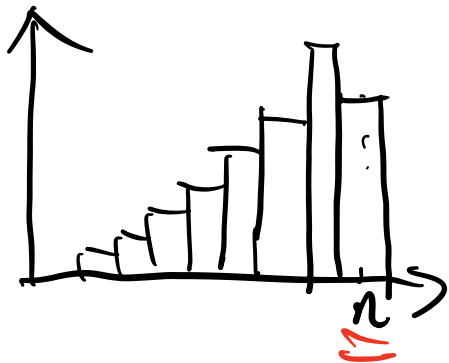
当 n 不变, p 变大时.

$\mu = E(X) = np \uparrow$ 图形右移.

$\text{Std}(X) = \sqrt{np(1-p)}$ 当 $p \rightarrow 0.5$ 时, $\text{Std}(X)$ 变大.

当 p 远离 0.5 时, $\text{Std}(X)$ 变小.

当 $p \rightarrow 1$ 时, 且 $n(1-p) \neq 10$ 时.



图右截断 $\Rightarrow$ skew to left.

当 $p \rightarrow 0$ 时. 且 np 不为 0 时.



左截断, $\Rightarrow$ skew to right.

!!! X 定义为成功的次数.

- 1) fixed total number of trials n .
- 2). only 2 outcomes each trial.
- 3). fixed success prob. p .
- 4). independent between trial.

Geometric R. V. distribution.

X represents the order of first success among a series of trials.

1). trials with only 2 outcomes
success / fail.

$X \sim \text{Geom}(p)$.

2) equal success prob. for all trials.

3). independent trials that do not affect each other.

PDF of geometric R.V.

$$P(X=k) = (1-p)^{k-1} p.$$

CDF of geometric R.V.

$$P(X \leq k) = P(X=1) + P(X=2) + \dots + P(X=k).$$

For geometric $1 \leq X < \infty$

$$E(X) = \frac{1}{p} \quad \text{Std}(X) = \sqrt{\frac{1-p}{p^2}}$$

shape: skew to the
right.

