Topic: Random Variables

1. If a probability distribution is symmetric, which of the following statements must be true?

   A. The distribution is normal.
   B. The distribution is uniform.
   C. The distribution is bimodal.
   D. The mean of the distribution is equal to the median of the distribution.
   E. The interquartile range of the distribution is equal to the standard deviation of the distribution.

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3. For a certain dog breed, the number of puppies in a litter typically varies from 2 to 6 . The following table shows the probability distribution of the random variable N, where N represents the number of puppies in a litter. Also shown are the squared deviations, or distances, from the expected value of 4.5 for the distribution.

Number of puppies	2	3	4	5	6
Squared deviation	6.25	2.25	0.25	0.25	2.25
Probability	0.05	0.15	0.25	0.35	0.20

What is the variance of the distribution?

$(x-\mu_x)^2$

$6.25 \times 0.05 + 2.25 \times 0.15 + \cdots + 2.25 \times 0.2$

   A. 1.12
   B. 1.25
   C. 1.41
   D. 1.58
   E. 2.25

$x_i - \mu_x$ → deviation.

$(x_i - \mu_x)^2$ → squared deviation.

$\sum (x_i - \mu_x)^2 p_i$ → variance.

$\sqrt{\sum (x_i - \mu_x)^2 p_i}$ → standard deviation

$E(aX+bY+c)$

$= aE(X)+bE(Y)+c.$

$Var(aX+bY+c)$

$= Var(aX)+Var(bY)$

$= a^2Var(X)+b^2Var(Y).$

$Std(aX+bY+c = \sqrt{a^2Std^2(X)+b^2Std^2(Y)}$

$E(aX)=aE(X)$

$Var(aX)=a^2Var(X)$

$Std(aX)=|a|Std(X)$

a 倍的 X.

$$Y = \underbrace{X_1 + X_2 + \cdots + X_a}_{a\text{个}X}$$

a个X · same μ_X, same σ_X

X同分布.

$$E(Y) = aE(X) = a\mu_X$$

$$Std(Y) = \sqrt{a}\, Std(X) = \sqrt{a}\,\sigma_X$$

4. Data were collected on the ages, in years, of the men and women enrolled in a large sociology course. Let the random variables M and W represent the ages of the men and women, respectively. The distribution of M has mean 20.7 years and standard deviation 1.73 years. The distribution of W has mean 20.2 years and standard deviation 1.60 years. Of all of those enrolled in the course, 54 percent are men and 46 percent are women. What is the mean age of the combined distribution of both men and women in the course?

   A. 20.2 years
   B. 20.43 years
   C. 20.45 years
   D. 20.47 years
   E. 40.9 years

   Topic: Linear Transformations

5. At a certain company, loan agents are paid based on the number of loans they close in a day. Based on company records, the number of loans X that a randomly selected loan agent closes on a randomly selected day has the probability distribution below.

x	1	2	3	4	5	6	7
$P(x)$	0.05	0.10	0.22	0.30	0.18	0.12	0.03

   At the company, the daily salary of a loan agent is \$150 plus \$50 per loan closed. Let Y represent the amount of money made by a randomly selected loan agent on a randomly selected day. Which of the following statements is NOT true?

   A. The mean of X is less than the mean of Y.
   B. The standard deviation of Y is approximately \$71.
   C. The mean daily salary is greater than \$350 per day.
   D. The standard deviation of X is less than the standard deviation of Y.
   E. The shape of the probability distribution of Y is unimodal and roughly symmetric.

6. In a certain computer card game, the player is awarded 5 points for each card that is moved to a correct position. The player is penalized 10 points for each minute the game is played. Let the random variable X represent the number of cards moved to a correct position, and let the random variable Y represent the number of minutes the game is played. The means and standard deviations of the random variables for a particular player are shown in the table below.

$$E(Z) = 5E(X) - 10E(Y)$$

$$= 5 \times 9.5 - 10 \times 5.4$$

$$Std(Z) = \sqrt{5^2 Std^2(X) + (-10)^2 Std^2(Y)}$$

$$= \sqrt{25 \times 12.9^2 + 100 \times 1.1^2}$$

Variable	Mean	Standard Deviation
X	9.5	12.9
Y	5.4	1.1

Assume that X and Y are independent. What are the expected value and the standard deviation of the points per game for the player?

A. The expected value is -6.5, and the standard deviation is 63.5.
B. The expected value is -6.5, and the standard deviation is 65.4.
C. The expected value is 4.1, and the standard deviation is 63.5.
D. The expected value is 4.1, and the standard deviation is 65.4.
E. The expected value is 101.5, and the standard deviation is 63.5.

7. Consider a data set of positive values, at least two of which are not equal. Which of the following sample statistics will be changed when each value in this data set is multiplied by a constant whose absolute value is greater than 1 ?

I. The mean

II. The median

III. The standard deviation

A. I only
B. II only
C. III only
D. I and II only
E. I, II and III

8. The random variable X is normally distributed with mean 5 and standard deviation 25 . The random variable Y is defined by $Y = 2 + 4X$. What are the mean and the standard deviation of Y ?

A. The mean is 20 and the standard deviation is 102 .
B. The mean is 20 and the standard deviation is 50 .
C. The mean is 22 and the standard deviation is 102 .
D. The mean is 22 and the standard deviation is 100 .
E. The mean is 22 and the standard deviation is 50 .

9. The random variable X has mean 12 and standard deviation 3. The random variable W is defined as $W = 7 + 2X$. What are the mean and standard deviation of W ?

A. The mean is 24, and the standard deviation is 6 .

B. The mean is 24 , and the standard deviation is 13 .

C. The mean is 31 , and the standard deviation is 3 .

D. The mean is 31 , and the standard deviation is 6 .

E. The mean is 31, and the standard deviation is 13 .

10. At a certain bakery, the price of each doughnut is \$1.50. Let the random variable D represent the number of doughnuts a typical customer purchases each day. The expected value and variance of the probability distribution of D are 2.6 doughnuts and 3.6 (doughnuts)^2, respectively. Let the random variable P represent the price of the doughnuts that a typical customer purchases each day. Which of the following is the standard deviation, in dollars, of the probability distribution of P ?

A. $1.5(3.6)$

B. $1.5\sqrt{3.6}$

C. $\sqrt{1.5(3.6)}$

D. $1.5(2.6)$

E. $1.5\sqrt{2.6}$

$SD(P) = SD(1.5D) = 1.5\,SD(D)$

$P = 1.5D$ $= 1.5\sqrt{3.6}$

11. A company sells concrete in batches of 5 cubic yards. The probability distribution of X, the number of cubic yards sold in a single order for concrete from this company, is shown in the table below.

X = the number of cubic yards	10	15	20	25	30
Probability	0.15	0.25	0.25	0.30	0.05

The expected value of the probability distribution of X is 19.25 and the standard deviation is 5.76. There is a fixed cost to deliver the concrete. The profit Y, in dollars, for a particular order can be described by $Y = 75X - 100$. What is the standard deviation of Y ?

A. \$332.00

B. \$432.00

C. \$532.00

D. \$1,343.75

E. \$1,400.00

Topic: Combining Random Variables

$C \sim N$ $\quad C = A - B$

$$Std(C) = \sqrt{Std^2(A) + Std^2(B)}$$

12. The distribution of random variable R has mean 10 and standard deviation 4. The distribution of random variable S has mean 7 and standard deviation 3. If R and S are independent, what are the mean and standard deviation of the distribution of $R - S$?

    A. Mean 3 and standard deviation 1
    B. Mean 3 and standard deviation 5
    C. Mean 3 and standard deviation 7
    D. Mean 17 and standard deviation 1
    E. Mean 17 and standard deviation 5

13. Two friends, Andy and Bob, participate in a game of bowling every week. From past experiences, it is known that both friends' scores are approximately normally distributed, where Andy has a mean score of 150 with a standard deviation of 30 , and Bob has a mean score of 165 with a standard deviation of 15 . Assuming that their scores are independent, which of the following values is closest to the probability that Andy will have a greater score than Bob in a single game?

    A. 0.16
    B. 0.28
    C. 0.31
    D. 0.33
    E. 0.37

14. Sean and Evan are college roommates who have part-time jobs as servers in restaurants. The distribution of Sean's weekly income is approximately normal with mean \$225 and standard deviation \$25. The distribution of Evan's weekly income is approximately normal with mean \$240 and standard deviation \$15. Assuming their weekly incomes are independent of each other, which of the following is closest to the probability that Sean will have a greater income than Evan in a randomly selected week?

    A. 0.067
    B. 0.159
    C. 0.227
    D. 0.303
    E. 0.354

15. Let X be a random variable whose values are the number of dots that appear on the uppermost face when a fair die is rolled. The possible values of X are $1, 2, 3, 4, 5$, and 6. The mean of X is $7/2$ and the variance of X is $35/12$. Let Y be the random variable whose value is the difference (first minus second) between

the number of dots that appear on the uppermost face for the first and second rolls of a fair die that is rolled twice. What is the standard deviation of Y ?

A. $\sqrt{\frac{35}{12}}$

B. $\sqrt{\frac{35}{12}} + \sqrt{\frac{35}{12}}$

C. $\sqrt{\frac{35}{12} + \frac{35}{12}}$

D. $\sqrt{\frac{35}{12} - \frac{35}{12}}$

E. $\frac{35}{12} + \frac{35}{12}$

16. A random variable X has a mean of 120 and a standard deviation of 15 . A random variable Y has a mean of 100 and a standard deviation of 9 . If X and Y are independent, approximately what is the standard deviation of $X - Y$?

A. 24.0

B. 17.5

C. 12.0

D. 6.0

E. 4.9

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19. High school students from track teams in the state participated in a training program to improve running times. Before the training, the mean running time for the students to run a mile was 402 seconds with standard deviation 40 seconds. After completing the program, the mean running time for the students to run a mile was 368 seconds with standard deviation 30 seconds. Let X represent the running time of a randomly selected student before training, and let Y represent the running time of the same student after training. Which of the following is true about the distribution of $X - Y$?

    A. The variables X and Y are independent; therefore, the mean is 34 seconds and the standard deviation is 10 seconds.

    B. The variables X and Y are independent; therefore, the mean is 34 seconds and the standard deviation is 50 seconds.

    C. The variables X and Y are not independent; therefore, the standard deviation is 50 seconds and the mean cannot be determined with the information given.

    D. The variables X and Y are not independent; therefore, the mean is 34 seconds and the standard deviation cannot be determined with the information given.

    E. The variables X and Y are not independent; therefore, neither the mean nor the standard deviation can be determined with the information given.