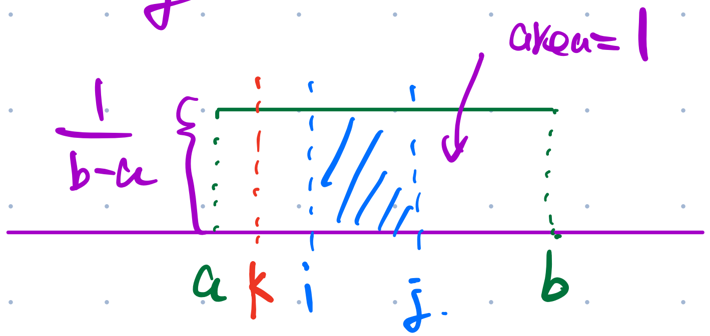
Continuous R.V. Distribution.

1) uniform distribution 均匀分布.

density curve:



$$P(i \le X \le j) = (j-i) \times \frac{1}{b-a} = \frac{j-i}{b-a}$$

for all continuous R.V.

$$P(X=k) = 0$$

$$E(X) = \frac{a+b}{2}$$

For continuos Random Variable.

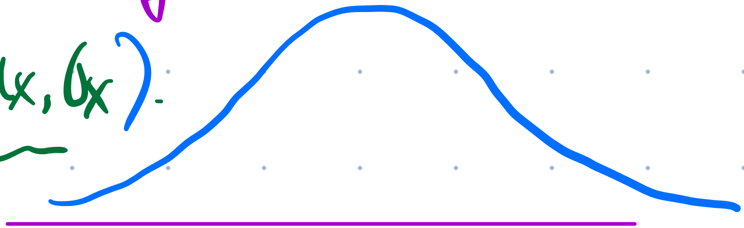
possible outcomes is infinite.

↪ need to use geometric probability model.

↪ need to calc the area of event.

2) Normal distribution

density curve.



symmetric

bell-shaped.

$X \sim N(\mu_x, \sigma_x)$.

$P(a \le X \le b) = \text{normalcdf}$

lower: a
upper: b
μ: μ_x
σ: σ_x

Central Limit Theorem

中心极限定理.

当 $Y = \underbrace{X_1 + X_2 + \cdots + X_n}_{n\text{个}X\text{之和}}$.

1) assume all X_i are → independent identically distributed.

当 $n \geq 30$ 时. $\boxed{Y \sim N}$

$\boxed{E(Y)} = E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots = nE(X)$

$\boxed{Std(Y)} = Std(\underbrace{X_1 + \cdots + X_n}_{n\text{个}X}) = \sqrt{Std^2(X_1) + Std^2(X_2) + \cdots + Std^2(X_n)}$

$= \sqrt{n\, Std^2(X)} = \sqrt{n}\, Std(X)$

记

$$E(aX+bY+cZ+d) = aE(X)+bE(Y)+cE(Z)+d$$

$$\mathrm{std}(aX+bY+cZ+d) = \sqrt{a^2\,\mathrm{std}^2(X)+b^2\,\mathrm{std}^2(Y)+c^2\,\mathrm{std}^2(Z)}$$

Tu记

mean of normal distribution

$$\mathrm{pdf} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

std of normal distribution

当 $Y = X_1 + \cdots + X_n$ ← 样本.

$Y \sim N$. $E(Y) = nE(X)$. $Std(Y) = \sqrt{n}\, Std(X)$.

$$\bar{X} = \frac{Y}{n} = \frac{X_1 + \cdots + X_n}{n}$$ sample mean.

↳ $Y \sim N$ 则 $\bar{X} \sim N$.

↳ $E(\bar{X}) = E\left(\frac{Y}{n}\right) = \frac{nE_X}{n} = E(X)$

样本均值的中心 Population 的中心.

$$Std(\bar{X}) = Std\left(\frac{Y}{n}\right) = \boxed{\frac{1}{n}} Std(Y) = \boxed{\frac{1}{n}} \sqrt{n}\, Std(X)$$

$$\text{std}(\bar{X}) = \frac{\text{std}(X)}{\sqrt{n}}$$