Topic: Conditional Probability

1. Carly commutes to work, and her commute time is dependent on the weather. When the weather is good, the distribution of her commute times is approximately normal with mean 20 minutes and standard deviation 2 minutes. When the weather is not good, the distribution of her commute times is approximately normal with mean 30 minutes and standard deviation 4 minutes. Suppose the probability that the weather will be good tomorrow is 0.9. Which of the following is closest to the probability that Carly's commute time tomorrow will be greater than 25 minutes?

   A. 0.0056
   B. 0.0894
   C. 0.0950
   D. 0.8055
   E. 0.9006

2. A contestant's final winnings on a game show are determined by a random selection of a base amount and a possible multiplier. For the base amount, the contestant randomly selects one of four cards, where two cards are marked $\$1,000$, one card is marked $\$2,000$, and one card is marked $\$5,000$. After choosing the card, the contestant randomly selects one of five chips, where three chips are red and two chips are white. If the selected chip is red, the contestant's final winnings are twice the value of the base amount. If the selected chip is white, the contestant's final winnings are the value of the base amount. What is the probability that a contestant's final winnings are exactly $\$2,000$?

   A. 0.100
   B. 0.200
   C. 0.250
   D. 0.325
   E. 0.400

3. A tropical storm is classified as major if it has sustained winds greater than 110 miles per hour. Based on data from the past two decades, a meteorologist estimated the following percentages about future storms.

   - 20% of all tropical storms will originate in the Atlantic Ocean, of which 20% will be classified as major.
   - 30% of all tropical storms will originate in the eastern Pacific Ocean, of which 15% will be classified as major.

- 50% of all tropical storms will originate in the western Pacific Ocean, of which 25% will be classified as major.

Based on the meteorologist's estimates, approximately what is the probability that a future tropical storm will originate in the Pacific Ocean and be classified as major?

A. 0.045
B. 0.125
C. 0.170
D. 0.400
E. 0.960

4. Mating eagle pairs typically have two baby eagles (called eaglets). When there are two eaglets, the parents always feed the older eaglet until it has had its fill, and then they feed the younger eaglet. This results in an unequal chance of survival for the two eaglets. Suppose that the older eaglet has a 50 percent chance of survival. If the older eaglet survives, the younger eaglet has a 10 percent chance of survival. If the older eaglet does not survive, the younger eaglet has a 30 percent chance of survival. Let X be the number of eaglets that survive. Which of the following tables shows the probability distribution of X ?

A.

x	0	1	2
$p(x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

B.

x	0	1	2
$p(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

C.

x	0	1	2
$p(x)$	0.35	0.60	0.05

D.

x	0	1	2
$p(x)$	0.05	0.90	0.05

E.

x	0	1	2
$p(x)$	0.10	0.30	0.50

5. The seniors at three high schools were surveyed about their plans after graduation. The following table shows the responses, classified by high school.

	Work	Military	College	Undecided	Total
High School A	99	49	138	63	349
High School B	62	26	156	54	298
High School C	83	31	124	71	309
Total	244	106	418	188	956

One senior from the high schools will be selected at random. What is the probability that the senior selected will not be from High School B given that the senior responded with a choice other than college?

A. $\frac{156}{418}$
B. $\frac{538}{956}$
C. $\frac{262}{418}$
D. $\frac{658}{956}$
E. $\frac{396}{538}$

6. Students at a local elementary school were shown a painting and asked which emotion-joy, happiness, love, or anger-they felt by looking at the painting. The students were classified by their age. The following table summarizes the responses of the students by age-group.

	Joy	Happiness	Love	Anger	Total
6 to 8 years	28	20	40	18	106
9 to 11 years	61	25	80	60	226
Total	89	45	120	78	332

One student from the school will be selected at random. What is the probability that the student is in the age-group of 6 to 8 years given that the ~~selected~~ student responded joy?

$P(6\text{–}8 \mid \text{Joy}) \quad \frac{28}{89}$

A. $\frac{89}{332}$
B. $\frac{28}{332}$
C. $\frac{28}{106}$
D. $\frac{106}{332}$
E. $\frac{28}{89}$

7. The table shows the responses from 103 people when asked if they support a proposal to expand the public library.

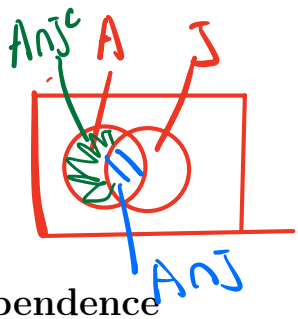
	Under the Age of 55	Age 55 or Older	Total
Yes	17	8	25
No	42	36	78
Total	59	44	103

One person from those who responded will be selected at random. Which of the following is closest to the probability that the person selected will be someone who responded no, given that the person selected is age 55 or older?

A. 0.350
B. 0.427
C. 0.462
D. 0.757
E. 0.818

8. At a small company, 40 percent of the employees are classified as junior level, and 32 percent of the employees work in department A. Of those classified as junior level, 55 percent work in department A. One employee will be selected at random. What is the probability that the selected employee works in department A and is not classified as junior level?

A. 0.10
B. 0.19
C. 0.22
D. 0.32
E. 0.50



$P(A \cap J^c)$

$P(A) = 0.32$ $P(J) = 0.4$

$P(A|J) = 0.55$

$P(A \cap J) = P(J) \cdot P(A|J) = 0.4 \times 0.55 = 0.22$

$P(A \cap J^c) = P(A) - P(A \cap J) = 0.32 - 0.22 = 0.1$

Topic: Independence

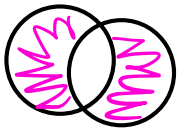
9. Events D and E are independent, with $P(D) = 0.6$ and $P(D \text{ and } E) = 0.18$. Which of the following is true?

A. $P = 0.12$
B. $P = 0.4$
C. $P(D \text{ or } E) = 0.28$
D. $P(D \text{ or } E) = 0.72$
E. $P(D \text{ or } E) = 0.9$

10. A company is considering purchasing the mineral rights to two different mountains. The probability that it will purchase the mineral rights to the first mountain is 0.55. The probability that it will purchase the mineral rights to the second mountain is 0.4. Assuming the decisions to purchase the mineral rights to each mountain are made independently, what is the probability that it will purchase the mineral rights to exactly one of the two mountains?

A. 0.18
B. 0.22
C. 0.33
D. 0.51

$P(B|A) = P(B)$



$P(A \cup B) - P(A \cap B)$

$P(A) + P(B) - 2P(A \cap B)$

$P(A \cap B) = P(A) \cdot P(B)$

$2(0.55 \times 0.4) + 0.55 + 0.4$

E. 0.95

11. A complex electronic device contains three components, A, B, and C. The probabilities of failure for each component in any one year are 0.01, 0.03, and 0.04, respectively. If any one component fails, the device will fail. If the components fail independently of one another, what is the probability that the device will not fail in one year?

A. Less than 0.01
B. 0.078
C. 0.080
D. 0.922
E. Greater than 0.99

12. Ms. Tucker travels through two intersections with traffic lights as she drives to the market. The traffic lights operate independently. The probability that both lights will be red when she reaches them is 0.22. The probability that the first light will be red and the second light will not be red is 0.33. What is the probability that the second light will be red when she reaches it?

A. 0.40
B. 0.45
C. 0.50
D. 0.55
E. 0.60

13. Joe and Matthew plan to visit a bookstore. Based on their previous visits to this bookstore, the probability distributions of the number of books they will buy are given below.

Number of books Joe will buy	0	1	2
Probability	0.50	0.25	0.25

Number of books Matthew will buy	0	1	2
Probability	0.25	0.50	0.25

Assuming that Joe and Matthew make their decisions independently, what is the probability that they will purchase no books on this visit to the bookstore?

A. 0.0625
B. 0.1250
C. 0.1875
D. 0.2500
E. 0.7500

$P(M+J) = 0$

$P[(M=0) \cap (J=0)]$

0.5×0.25

14. The distribution of colors of candies in a bag is as follows.

Color	Brown	Red	Yellow	Green	Orange
Proportion	0.3	0.2	0.2	0.2	0.1

If two candies are randomly drawn from the bag with replacement, what is the probability that they are the same color?

A. 0.09
B. 0.22
C. 0.25
D. 0.75
E. 0.78

15. Ali surveyed 200 students at a school and recorded the eye color and the gender of each student. Of the 80 male students who were surveyed, 60 had brown eyes. If eye color and gender are independent, how many female students surveyed would be expected to have brown eyes?

$P(B|F) = ?$

$P(B|M) = \frac{60}{80}$

$P(B|F) = P(B)$

$P(B|M) = P(B)$

$= \frac{60}{80}$

$120 \times \frac{60}{80}$

A. 5
B. 20
C. 30
D. 90
E. 100

16. The SC Electric Company has bid on two electrical wiring jobs. The owner of the company believes that

- the probability of being awarded the first job (event A) is 0.75;
- the probability of being awarded the second job (event B) is 0.5; and
- the probability of being awarded both jobs (event (A and B)) is 0.375.

If the owner's beliefs are correct, which of the following statements must be true concerning event A and event B ?

A. Event A and event B are mutually exclusive and are independent.

B. Event A and event B are mutually exclusive and are not independent.

C. Event A and event B are not mutually exclusive and are independent.

D. Event A and event B are not mutually exclusive and are not independent.

E. Event A and event B are not mutually exclusive, and independence cannot be determined with the information given.

17. For which of the following probability assignments are events A and B independent?

① $P(B|A) = P(B)$ ② $P(A \cap B) = P(A) \cdot P(B)$

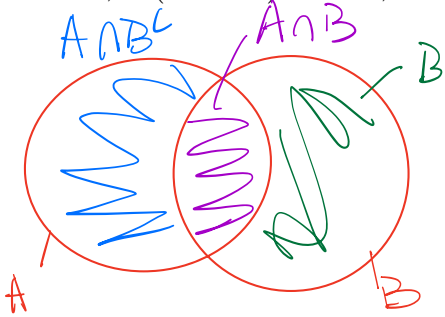
A. $P(A \cap B^c) = 0.3$, $P(A \cap B) = 0.12$, and $P(A^c \cap B) = 0.4$.

B. $P(A \cap B^c) = 0.3$, $P(A \cap B) = 0.3$, and $P(A^c \cap B) = 0.3$.

C. $P(A \cap B^c) = 0.1$, $P(A \cap B) = 0.1$, and $P(A^c \cap B) = 0.4$.

D. $P(A \cap B^c) = 0.3$, $P(A \cap B) = 0.0$, and $P(A^c \cap B) = 0.2$.

E. $P(A \cap B^c) = 0.5$, $P(A \cap B) = 0.1$, and $P(A^c \cap B) = 0.4$.



$P(A) = P(A \cap B^c) + P(A \cap B)$

$P(B) = P(A^c \cap B) + P(A \cap B)$

Prob. of 1-event

1). classic prob. model.

outcome 同机会

2) Geometric prob. model.

Outcome 不同机会.

Outcome 无限多.

Prob. of 2-event.

2 formulas.

2 tools

2 special relationship { Independent / Mutually Exclusive.

Interpretation