

Inference for population proportion p .

想推测全校学生中, 男生的占比.

猜一个 $\sqrt{p}$ 的大致范围

p 大致在 $(0.45, 0.48)$

Confidence Interval

做推断就是算“概率”

对 p 的某 claim 进行验证

$p > 0.5$ 是不对的

Hypothesis Testing.

Hypothesis Testing for P.

*要做 Hypothesis testing. 先满足条件.

Settings: eg. 别人对全校学生男性占比

提出个一个想法 or claim. $p > 0.5$

你想验证这个想法是否有
证据支撑.

所以, 你去抽了一个 SRS 的样本.

$n=50$

构建了 Hypothesis.

原假设 ^{null} hypo. $H_0: P=0.5 \leftarrow$ 等于.

alternative hypothesis. $H_A: P > 0.5 \leftarrow <, > \text{ or } \neq.$

null hypothesis $\leftarrow$ 假定已成立,
看看能不能推翻

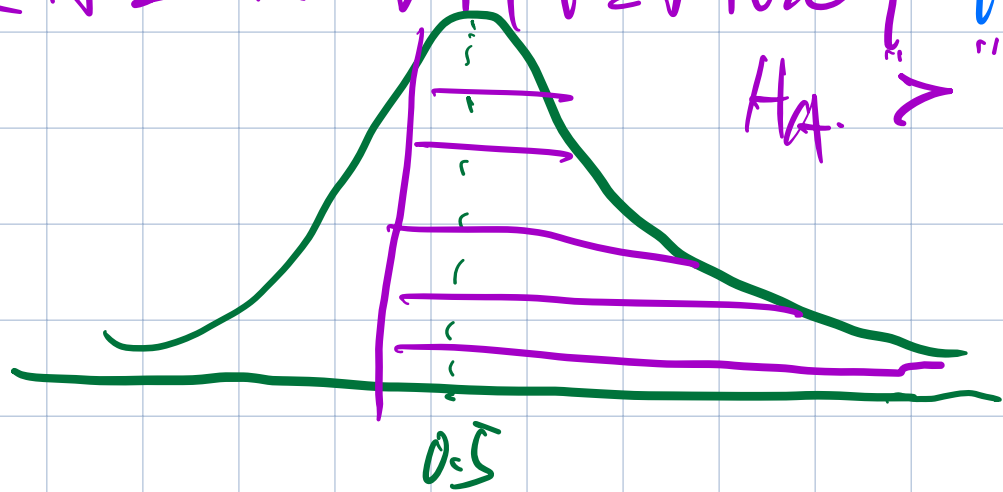
$p=0.5$ 已成立.

$\Rightarrow \hat{p} \sim N(0.5, \sqrt{\frac{0.5(1-0.5)}{50}})$

Scenario ①

我们取到的样本是 $\hat{p}=0.48$

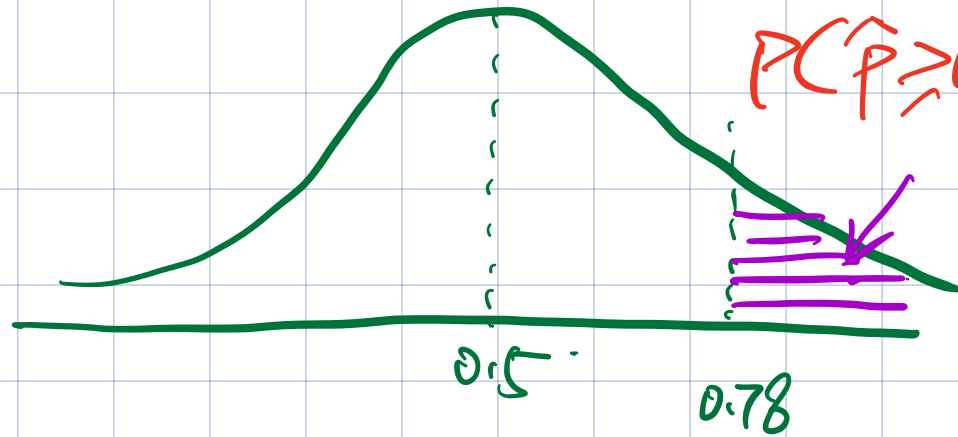
算取到此样本以及比取到此
样本更符合 H_a 的样本的概率 $p\text{-value}$



$p > 0.5 \Rightarrow$ 取到此样本是个大概率

无法推翻 $H_0 \leftarrow$ 事件. 没什么好奇怪的

Scenario (2) 抽到样本 $\hat{p} = 0.78$



$$P(\hat{p} \geq 0.78) = 0.023$$

推翻 H_0
支持 H_1 .

抽到一个小概率样本 $\Rightarrow$ 见鬼了

日常情况见不到鬼, 有猫月戒.

原假设不符合情况!! 所以导致见鬼!

① 定 Hypothesis. H_0 $p = ?$

H_a $p < \text{or} > \text{or} \neq ?$

② 验 3 conditions.

1) SRS.

2) 10%

3) Large count.

$\downarrow$
 $\uparrow$
 $P \sim N:$

$\downarrow$

③ 算 p -value.

④ 做判断

$p\text{-value} > 0.05$
 H_0 成立.

$p\text{-value} < 0.05$
 H_a 成立.

State ① 解决什么问题. 做 H.T.

② 用什么变量. P is -----

Plan. ④ 想用什么方法.

⑤ 用此方法是否符合条件

Do ① 记等关键指标.

Conclude ① 做出结论.

Hypothesis test. 有claim 验证对不对.

Confidence
一个大致范围.

Interval 求! pop. parameter.

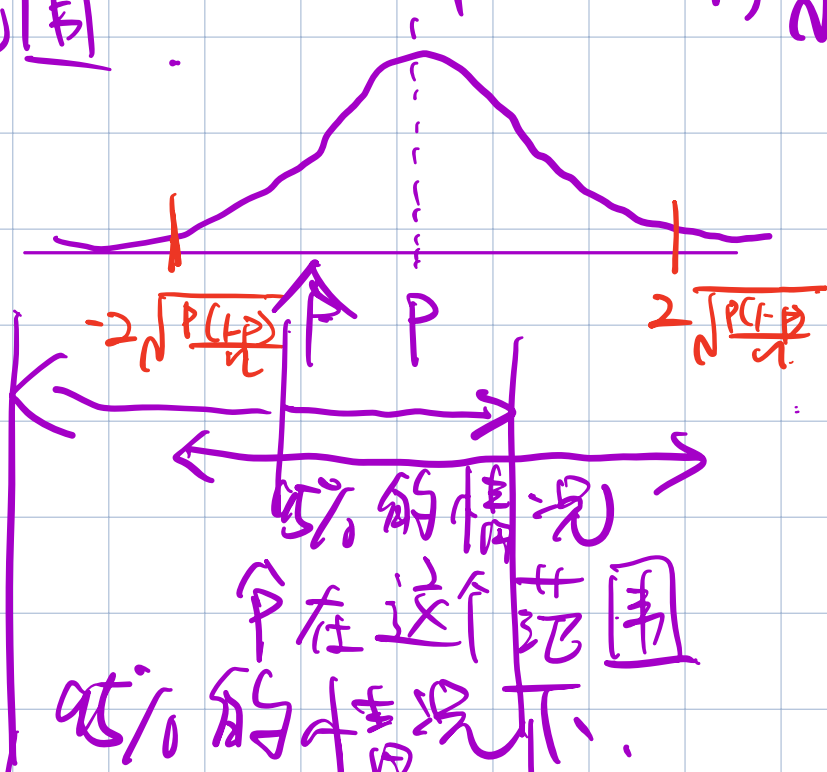
全体参数.

全体均值 μ

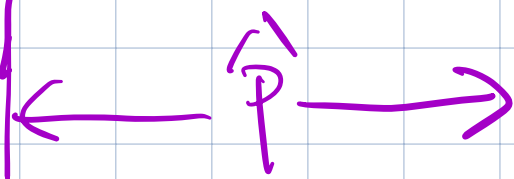
全体百分比 P .

$\hat{p} = \frac{9}{20} = 0.45$ $n=20$ 问全校的 p 在什么范围

$\hat{p} \sim N(p, \sqrt{\frac{p(1-p)}{n}})$



在这个范围



$$-2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$+2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

95%的情况 $(\hat{p} - 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$

包含 P . $(0.227, 0.672)$

$$\frac{1.344}{1.344} \Rightarrow \frac{0.344}{1.344}$$

$$p = 70\% = 0.7$$

控制半径.

$$\approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < 0.03.$$

$$\frac{2\sqrt{\hat{p}(1-\hat{p})}}{0.03} < \sqrt{n}$$

$$\sqrt{n} > \frac{2\sqrt{\hat{p}(1-\hat{p})}}{0.03}$$

$$n > \frac{2}{0.03^2} \left[\hat{p}(1-\hat{p}) \right]$$

$\hat{p} = 0.5$ 时, $\hat{p}(1-\hat{p})$ 达最高值

$$n > \frac{2}{0.03^2} \times 0.5 \times 0.5$$

$$n > 555.55$$

confidence interval $\rightarrow$ 区间. C.I.

区间中心 sample statistics. $\hat{p}, \bar{x}, b$

区间半径. Margin of Error MOE

$$\text{Margin of Error} = \text{Critical Value} \times \text{Standard Error.}$$

置信度

Confidence Level. 95%

Confidence level 不是概率。

$$C.L = 95\% \Rightarrow CV = 1.96$$

Standard Error

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Sample (值)

The diagram illustrates the components of the standard error formula. The text 'Standard Error' is written in pink. The formula $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ is written in pink, with the square root symbol and the denominator 'n' highlighted. The text 'Sample (值)' is written in green below the formula. Four green arrows indicate the relationship: one points from 'Sample (值)' to 'n', another points from 'Sample (值)' to the denominator of the fraction inside the square root, a third points from 'Sample (值)' to the square root symbol, and a fourth points from 'Standard Error' to the entire expression.

$n=60$ $\hat{p}=0.52$ $P=0.8?$ $P>0.8?$

1) 定假设.

2) 验条件.

3) 算 P -value

4) 做结论.

定假设.

Ta说: $P > 0.8$ or $P = 0.8$.

你觉得: $P < 0.8$

变成 H_A .

$H_A: P < 0.8$

将 $>$ or $<$ or $\neq$ 改成 " $=$ "
用作 H_0

$H_0: P = 0.8$

H_0 is called Null hypothesis. 原假设.
假定已经成立. 就把 $p=0.8$ 作为 pop. proportion.

⇒ 可构建 sampling distribution

$$\hat{p} \sim N(0.8, \sqrt{\frac{0.8 \times 0.2}{60}})$$

⇒ 看一看取到这样 ($\hat{p}=0.52$) 一个样本.
或比此样本更符合 H_A 方向样本 ($\hat{p} < 0.52$)
的概率. → p-value.

如果 p -value 很大 ($p\text{-value} > 0.05$)

⇒ 很轻松取到这样样本, 没什么大不了的.

⇒ 平常情况, 无法推翻 H_0 .

如果 p -value 很小 ($p\text{-value} < 0.05$)

⇒ 取到这样样本非常不可能

⇒ 你取到了这样的样本, 见鬼了.

⇒ 不该见鬼有东西出问题了.

⇒ 原假设出问题了.

⇒ 推翻原假设 H_0 ($p = 0.05$)

计算

$$\hat{p} \sim N(0.8, \sqrt{\frac{0.8 \times 0.2}{60}})$$

$$p\text{-value} = P(\hat{p} \leq 0.5) = \text{Normalcdf}$$

$$\left(\begin{array}{l} \text{lower: } 0 \\ \text{upper: } 0.5 \\ \mu: 0.8 \\ \sigma: \sqrt{\frac{0.8 \times 0.2}{60}} \end{array} \right)$$

$$\approx 0.$$

结论:

$p\text{-value} \approx 0 < 0.05 \therefore$ 拒绝 H_0 .

$\therefore H_A$ 成立.

4-step process

State $\leftarrow$ 目的. 目标. settings 定义

Plan $\leftarrow$ 用什么方法. 验证条件

Do $\leftarrow$ 关键指标计算

conclude. $\leftarrow$ 根据关键指标作出结论.

4-step for HT.

1> State: We are testing.

做的工作

$$H_0: P = 0.8.$$



$$H_A: P < 0.8$$



$$\text{at } \alpha = 0.05$$

细节



先定 H_A . 将 H_A 设置为你想证明的
再定 H_0 . H_A 是用 ">" or "<" or " $\neq$ "
 H_0 用 " $=$ ".

α or α -level 是你设定的小概率
的标准.

变量定义



Where P is the
true proportion of
female student among
all students in our
school.

Plan: 1) 声明所用方法. $\Rightarrow$ 验证方法适用条件

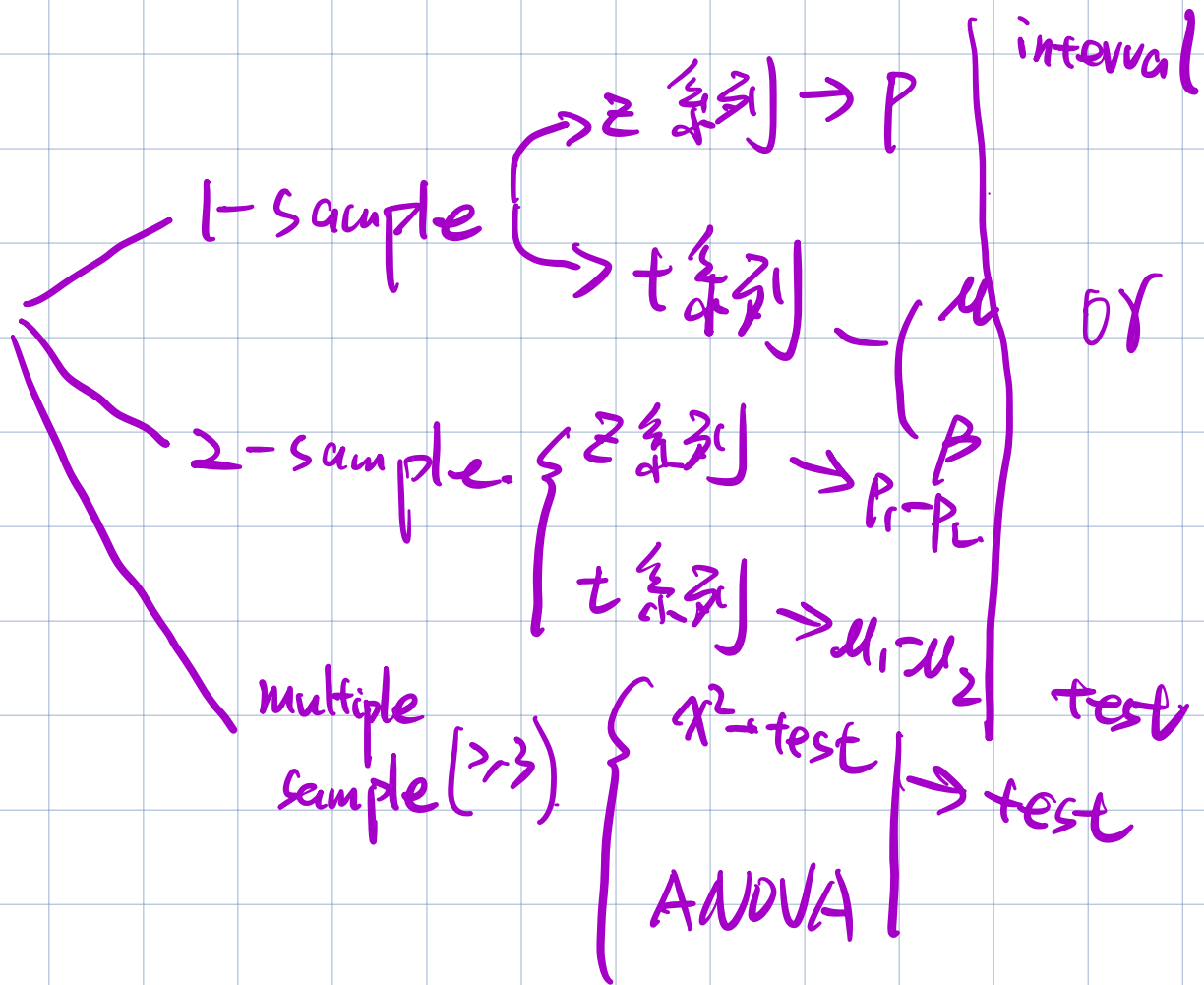
We are using 1-sample z -test for p .

其它情况 2-sample t test for μ_1, μ_2

1-sample 因为只有 p 不是 $p_1 - p_2$.

若是 $p_1 - p_2$ 则为 2-sample. 如例 $\mu_1 - \mu_2$ 是 2-sample
 z -test 中的 z 对应 for p . 若是 for μ 则是 t
原因是 sampling distribution 形状不同.

z -test 中的 test 对应 我们做的 hypothesis test.



验证条件: 3条件 (大部分)

1) SRS the sample of 60 students was randomly selected. ✓

(It is reasonable to assume)

2) 10% condition the sample size (60) is less than 10% of the population size (600) ✓

test for p 3) $n\hat{p}_0 = 60 \times 0.8 = 48 \geq 10$
 $n(1-\hat{p}_0) = 60 \times 0.2 = 12 \geq 10$ ✓

D_0 : 记算关键指标. (for test 计算 p-value)

用计算器. 写明参数

By using calculator $\swarrow$ H_0 里的 p 值

1-propZ test

函数名

$P_0: 0.8$

$x: \hat{p} \cdot n = 0.47 \times 60 \leftarrow$ 4个5入

$n: \text{sample size } 60$

$\text{prop: } "<"$

$H_1 \neq$ 不等号方向

We get p-value ≈ 0

$z = -6.4549$

Conclude:

Because our p -value (α_0) is less than our α -level (0.05). We can reject H_0 . We do have convincing evidence that the true proportion of female student among all student is less than 0.8

Type I Error :

	R	O	T
	↑	↑	↑
	Reject	H_0	
		H_0	True.

Probability of making type I error: α -level

Type II error : $\overline{H_0}$

fail to reject

H_0

Probability of Type II error.

$\rightarrow \beta$ -level.

H_0 false.

$= 1 - \text{Power of test.}$

Power of test $\leftarrow$

1) Sample size.

$n \uparrow \rightarrow \text{Power} \uparrow \rightarrow \beta \downarrow$

2) α -level

$\alpha \uparrow \rightarrow \text{Power} \uparrow \rightarrow \beta \downarrow$

3) effective gap.
 d

Confidence Interval

$$\left| \begin{array}{l} \downarrow P_0 - P \\ \downarrow \text{Power} \uparrow \rightarrow \beta \downarrow \end{array} \right.$$

找 pop. parameter 的大致范围

有 $\hat{P}$. 找 PEC > How?

→ 围绕 $\hat{P}$ 构建 alternative sampling distribution.

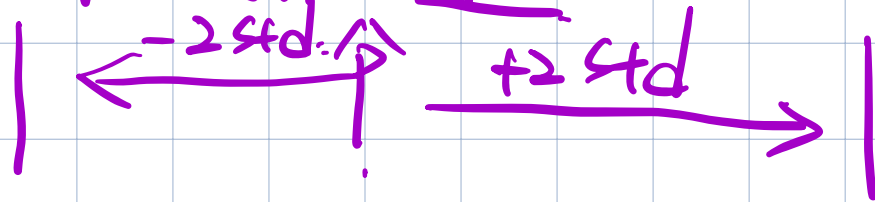
why ed alternative? $\hat{P}$ is real sampling dist.

real sampling distr: $\leftarrow$ via Pop. param
为 中心 -

alternative sampling distr $\leftarrow$ via sample
statistics

1) via $\hat{p}$ 为中心. via $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
为 std. 构建 alt-sampling
distr: $\tilde{p} \sim N(\hat{p}, \sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$
为 中心.

$\tilde{p}$ 是 p 的可能值



↑
lower bound of
confidence level

$$0.47 - \sqrt{\frac{0.47 \times 0.53}{60}}$$

0.341

0.47

↑
upper bound
of confidence
level

$$0.47 + 2 \times \sqrt{\frac{0.47 \times 0.53}{60}}$$

0.598

$P \in (0.341, 0.598)$

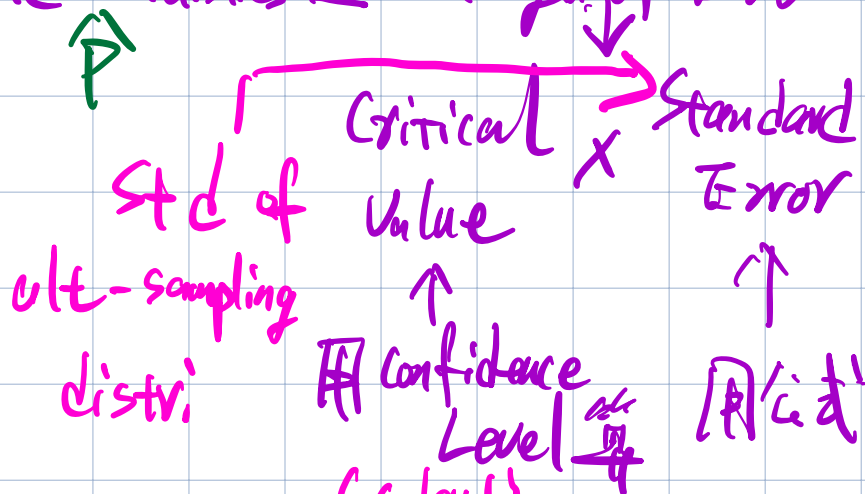


$RR \Rightarrow E(\hat{p}) = p \Rightarrow \hat{p}$ 和 p 应该很像.

$\hat{p}$ is the best guess for p .

Confidence interval 置信区间.

→ Sample Statistics $\pm$ Margin of Error



C-level 作为中间值 $\rightarrow$ Z 值.

$$\hat{p} \pm z\text{-critical} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$\uparrow$
题里给

$\uparrow$
由 α -level 算

$\uparrow$
公式算

State: We are constructing a confidence interval for $\rightarrow$ the true proportion of female students among all students in our school (p)

or $\rightarrow p$, where p is the true - our school.
at 95% confidence level.

Plan. ① We are using 1-sample z -interval for p . if all conditions are met

② 1) SRS. all 60 students in the sample were randomly selected. ✓

2) 10% (it is reasonable to assume that) the sample size (60) is small than 10% of it's population size (80) ✓

3) Large count condition

$$n\hat{p} = 60 \times 0.47 = 28.2 \geq 10$$

$$n(1-\hat{p}) = 60 \times (1-0.47) = 31.8 \geq 10 \quad \checkmark$$

Do.

By using calculator

$$\text{1-PropZint} \left(\begin{array}{l} x: 28 \\ n: 60 \\ \text{C-level: } 0.95 \end{array} \right)$$

We get confidence interval $(0.340, 0.593)$ at $\text{C-level} = 0.95$

Conclude We are 95% confident that the true proportion of female student among all students in our school is between 0.340 to 0.593.