1 Probability: General Rules, Independence, and Conditional Probability

1. A high school science teacher has 78 students. Of those students, 35 are in the band and 32 are on a sports team. There are 16 students who are not in the band or on a sports team. One student from the 78 students will be selected at random. Let event B represent the event of selecting a student in the band, and let event S represent the event of selecting a student on a sports team. Are B and S mutually exclusive events?

   A. No, because $P(B \cap S) = \frac{5}{78}$.
   B. No, because $P(B \cap S) = \frac{48}{78}$.
   C. No, because $P(B \cap S) = \frac{62}{78}$.
   D. Yes, because $P(B \cap S) = \frac{5}{78}$.
   E. Yes, because $P(B \cap S) = \frac{62}{78}$.

2.

	Team		
	Home	Away	Total
Purchased food	120	40	160
Did not purchase food	60	30	90
Total	180	70	250

The table shows data that were collected from people who attended a certain high school basketball game. A person who attended the game will be selected at random. Which of the following correctly interprets mutually exclusive events represented by the table?

A. Rooting for the home team and rooting for the away team

B. Rooting for the home team and purchasing food during the game

C. Rooting for the away team and purchasing food during the game

D. Rooting for the home team and not purchasing food during the game

E. Not rooting for the home team and not purchasing food during the game

3. A middle school chess club has 5 members: Adam, Bradley, Carol, Dave, and Ella. Two students from the club will be selected at random to participate in the county chess tournament. What is the probability that Adam and Ella will be selected?

   A. $\frac{1}{20}$
   B. $\frac{1}{10}$
   C. $\frac{1}{8}$
   D. $\frac{1}{7}$
   E. $\frac{1}{4}$

4. Events D and E are independent, with $P(D) = 0.6$ and $P(D \text{ and } E) = 0.18$. Which of the following is true?

A. $P(E) = 0.12$

B. $P(E) = 0.4$

C. $P(D \text{ or } E) = 0.28$

D. $P(D \text{ or } E) = 0.72$

E. $P(D \text{ or } E) = 0.9$

5. One student from a high school will be selected at random. Let A be the event that the selected student is a student athlete, and let B be the event that the selected student drives to school. If $P(A \cap B) = 0.08$ and $P(B \mid A) = 0.25$, what is the probability that the selected student will be a student athlete?

A. 0.02

B. 0.17

C. 0.32

D. 0.33

E. 3.13

6. Each of the faces of a fair six-sided number cube is numbered with one of the numbers 1 through 6, with a different number appearing on each face. Two such number cubes will be tossed, and the sum of the numbers appearing on the faces that land up will be recorded. What is the probability that the sum will be 4, given that the sum is less than or equal to 6?

   A. 2/36

   B. 3/36

   C. 3/15

   D. 2/9

   E. 4/6

7. As a promotion, the first 50 customers who entered a certain store at a mall were asked to choose from one of two discounts. The first discount choice was 20% off all purchases made that day. The second discount choice was 10% off all purchases for the week. Of those who received the discounts, 28 chose the first discount and 22 chose the second discount. One customer will be selected at random from those who received a discount. Let F represent the event that the selected person chose the first discount, and let S represent the event that the selected person chose the second discount. Are F and S mutually exclusive events?

   A. Yes, because $P(F \cap S) = 0$.

   B. Yes, because $P(F \cap S) = 0.12$.

   C. Yes, because $P(F \cap S) = 1$.

   D. No, because $P(F \cap S) = 0$.

   E. No, because $P(F \cap S) = 1$.

8. For flights from a particular airport in January, there is a 30 percent chance of a flight being delayed because of icy weather. If a flight is delayed because of icy weather, there is a 10 percent chance the flight will also be delayed because of a mechanical problem. If a flight is not delayed because of icy weather, there is a 5 percent chance that it will be delayed because of a mechanical problem. If one flight is selected at random from the airport in January, what is the probability that the flight selected will have at least one of the two types of delays?

A. 0.065

B. 0.335

C. 0.350

D. 0.450

E. 0.665

9. Ali surveyed 200 students at a school and recorded the eye color and the gender of each student. Of the 80 male students who were surveyed, 60 had brown eyes. If eye color and gender are independent, how many female students surveyed would be expected to have brown eyes?

A. 5

B. 20

C. 30

D. 90

E. 100

10. The SC Electric Company has bid on two electrical wiring jobs. The owner of the company believes that

- the probability of being awarded the first job (event A) is 0.75;
- the probability of being awarded the second job (event B) is 0.5; and
- the probability of being awarded both jobs (event (A and B)) is 0.375.

If the owner's beliefs are correct, which of the following statements must be true concerning event A and event B?

A. Event A and event B are mutually exclusive and are independent.

B. Event A and event B are mutually exclusive and are not independent.

C. Event A and event B are not mutually exclusive and are independent.

D. Event A and event B are not mutually exclusive and are not independent.

E. Event A and event B are not mutually exclusive, and independence cannot be determined.

11. For which of the following probability assignments are events A and B independent?

A. $P(A \cap B^c) = 0.3, P(A \cap B) = 0.12$, and $P(A^c \cap B) = 0.4$.

B. $P(A \cap B^c) = 0.3, P(A \cap B) = 0.3$, and $P(A^c \cap B) = 0.3$.

C. $P(A \cap B^c) = 0.1, P(A \cap B) = 0.1$, and $P(A^c \cap B) = 0.4$.

D. $P(A \cap B^c) = 0.3, P(A \cap B) = 0.0$, and $P(A^c \cap B) = 0.2$.

E. $P(A \cap B^c) = 0.5, P(A \cap B) = 0.1$, and $P(A^c \cap B) = 0.4$.

12. Ms. Tucker travels through two intersections with traffic lights as she drives to the market. The traffic lights operate independently. The probability that both lights will be red when she reaches them is 0.22. The probability that the first light will be red and the second light will not be red is 0.33. What is the probability that the second light will be red when she reaches it?

A. 0.40

B. 0.45

C. 0.50

D. 0.55

E. 0.60

13. In a certain school, 17 percent of the students are enrolled in a psychology course, 28 percent are enrolled in a foreign language course, and 32 percent are enrolled in either a psychology course or a foreign language course or both. What is the probability that a student chosen at random from this school will be enrolled in both a foreign language course and a psychology course?

A. 0.45

B. 0.32

C. 0.20

D. 0.13

E. 0.05

14. The probability that a new microwave oven will stop working in less than 2 years is 0.05. The probability that a new microwave oven is damaged during delivery and stops working in less than 2 years is 0.04. The probability that a new microwave oven is damaged during delivery is 0.10. Given that a new microwave oven is damaged during delivery, what is the probability that it stops working in less than 2 years?

A. 0.05

B. 0.06

C. 0.10

D. 0.40

E. 0.50

15. A student is applying to two different agencies for scholarships. Based on the student's academic record, the probability that the student will be awarded a scholarship from Agency A is 0.55 and the probability that the student will be awarded a scholarship from Agency B is 0.40. Furthermore, if the student is awarded a scholarship from Agency A, the probability that the student will be awarded a scholarship from Agency B is 0.60. What is the probability that the student will be awarded at least one of the two scholarships?

    A. 0.60

    B. 0.62

    C. 0.71

    D. 0.73

    E. 0.95

2 Discrete Random Variables

1. The following table shows the probability distribution for the number of books a student typically buys at the annual book fair held at an elementary school.

Number of Books	0	1	2	3	4	5	6	7
Probability	0.35	0.20	0.15	0.10	0.07	0.08	0.04	0.01

Let the random variable B represent the number of books a student buys at the next book fair. What is the expected value of B?

A. 0

B. 1.00

C. 1.79

D. 3.50

E. 28

2. At a large regional collegiate women's swim meet, an official records the time it takes each swimmer to swim 100 meters for all swimmers who compete in only one stroke category. The following table shows the mean times and corresponding standard deviations for the collegiate women at the swim meet for each of the four stroke categories.

Stroke Category	Mean **100 meter Time**	Standard Deviation
Backstroke	55.6 seconds	0.70 seconds
Breaststroke	63.3 seconds	0.92 seconds
Butterfly	54.4 seconds	0.94 seconds
Freestyle	50.2 seconds	0.76 seconds

For each of the 4 stroke categories, consider a random variable representing the time of a randomly selected swimmer in that category. What is the standard deviation of the sum of the 4 random variables?

A. 0.83 seconds

B. 1.67 seconds

C. 2.80 seconds

D. 3.32 seconds

E. 3.76 seconds

3. A player pays \$15 to play a game in which a chip is randomly selected from a bag of chips. The bag contains 10 red chips, 4 blue chips, and 6 yellow chips. The player wins \$5 if a red chip is selected, \$10 if a blue chip is selected, and \$20 if a yellow chip is selected. Let the random variable X represent the amount won from the selection of the chip, and let the random variable W represent the total amount won, where $W = X - 15$. What is the mean of W?

   A. \$10.50

   B. \$4.50

   C. -\$4.50

   D. -\$6.50

   E. -\$10.50

4. A city department of transportation studied traffic congestion on a certain highway. To encourage carpooling, the department will recommend a carpool lane if the average number of people in passenger cars on the highway is less than 2. The probability distribution of the number of people in passenger cars on the highway is shown in the table.

Number of people	1	2	3	4	5
Probability	0.56	0.28	0.08	0.06	0.02

Based on the probability distribution, what is the mean number of people in passenger cars on the highway?

A. 0.28

B. 0.56

C. 1.7

D. 2

E. 3

n倍的X.

当 $Y = nX$. $std(Y) = n\,std(X)$

n个 X_i

当 $Y = X_1 + \cdots + X_n$. X iid with μ_x, σ_x.

$\sigma_Y = \sqrt{n}\,\sigma_x$.

5. Every Thursday, Matt and Dave's Video Venture has "roll-the-dice" day. A customer may choose to roll two fair dice and rent a second movie for an amount (in cents) equal to the numbers uppermost on the dice, with the larger number first. Let X represent the amount paid for a second movie on roll-the-dice day. The expected value of X is \$0.47 and the standard deviation of X is \$0.15. If a customer rolls the dice and rents a second movie every Thursday for 30 consecutive weeks, what is the approximate probability that the total amount paid for these second movies will exceed \$15.00?

A. 0

B. 0.09

C. 0.14

D. 0.86

E. 0.91

= $\sqrt{30}$ x0.15 =

6. The distribution of random variable R has mean 10 and standard deviation 4. The distribution of random variable S has mean 7 and standard deviation 3. If R and S are independent, what are the mean and standard deviation of the distribution of $R - S$?

   A. Mean 3 and standard deviation 1

   B. Mean 3 and standard deviation 5

   C. Mean 3 and standard deviation 7

   D. Mean 17 and standard deviation 1

   E. Mean 17 and standard deviation 5

7. At a certain bakery, the price of each doughnut is \$1.50. Let the random variable D represent the number of doughnuts a typical customer purchases each day. The expected value and variance of the probability distribution of D are 2.6 doughnuts and 3.6 (doughnuts)^2, respectively. Let the random variable P represent the price of the doughnuts that a typical customer purchases each day. Which of the following is the standard deviation, in dollars, of the probability distribution of P?

   A. $1.5(3.6)$

   B. $1.5\sqrt{3.6}$

   C. $\sqrt{1.5(3.6)}$

   D. $1.5(2.6)$

   E. $1.5\sqrt{2.6}$

8. The XYZ Office Supplies Company sells calculators in bulk at wholesale prices, as well as individually at retail prices. The following tables are estimates for next year's sales.

WHOLESALE SALES

Number Sold	2,000	5,000	10,000	20,000
Probability	0.1	0.3	0.4	0.2

RETAIL SALES

Number Sold	600	1,000	1,500
Probability	0.4	0.5	0.1

What profit does XYZ Office Supplies Company expect to make for the next year if the profit from each calculator sold is \$20 at wholesale and \$30 at retail?

A. \$10,590

B. \$220,700

C. \$264,750

D. \$833,100

E. \$1,002,500

9. In a certain game, a fair die is rolled and a player gains 20 points if the die shows a "6." If the die does not show a "6," the player loses 3 points. If the die were to be rolled 100 times, what would be the expected total gain or loss for the player?

A. A gain of about 1,700 points

B. A gain of about 583 points

C. A gain of about 83 points

D. A loss of about 250 points

E. A loss of about 300 points

10. A box contains 10 tags, numbered 1 through 10, with a different number on each tag. A second box contains 8 tags, numbered 20 through 27, with a different number on each tag. One tag is drawn at random from each box. What is the expected value of the sum of the numbers on the two selected tags?

A. 13.5

B. 14.5

C. 15.0

D. 27.0

E. 29.0

11. Let X represent the number on the face that lands up when a fair six-sided number cube is tossed. The expected value of X is 3.5, and the standard deviation of X is approximately 1.708. Two fair six-sided number cubes will be tossed, and the numbers appearing on the faces that land up will be added. Which of the following values is closest to the standard deviation of the resulting sum?

A. 1.708

B. 1.848

C. 2.415

D. 3.416

E. 5.835

12. A company that makes fleece clothing uses fleece produced from two farms, Northern Farm and Western Farm. Let the random variable X represent the weight of fleece produced by a sheep from Northern Farm. The distribution of X has mean 14.1 pounds and standard deviation 1.3 pounds. Let the random variable Y represent the weight of fleece produced by a sheep from Western Farm. The distribution of Y has mean 6.7 pounds and standard deviation 0.5 pound. Assume X and Y are independent. Let W equal the total weight of fleece from 10 randomly selected sheep from Northern Farm and 15 randomly selected sheep from Western Farm. Which of the following is the standard deviation, in pounds, of W?

    A. $1.3 + 0.5$

    B. $\sqrt{1.3^2 + 0.5^2}$

    C. $\sqrt{10(1.3)^2 + 15(0.5)^2}$

    D. $\sqrt{10^2(1.3)^2 + 15^2(0.5)^2}$

    E. $\sqrt{\frac{1.3^2}{10} + \frac{0.5^2}{15}}$

3 The Normal Distribution

1. Two college roommates have each committed to donating to charity each week for the next year. The roommates' weekly incomes are independent of each other. Suppose the amount donated in a week by one roommate is approximately normal with mean \$30 and standard deviation \$10, and the amount donated in a week by the other roommate is approximately normal with mean \$60 and standard deviation \$20. Which of the following is closest to the expected number of weeks in a 52-week year that their combined donation will exceed \$120?

   A. 0; the combined donation never exceeds \$120 in a week

   B. 1 week

   C. 3 weeks

   D. 5 weeks

   E. 8 weeks

2. The distribution of random variable R has mean 10 and standard deviation 4. The distribution of random variable S has mean 7 and standard deviation 3. If R and S are independent, what are the mean and standard deviation of the distribution of $R - S$?

A. Mean 3 and standard deviation 1

B. Mean 3 and standard deviation 5

C. Mean 3 and standard deviation 7

D. Mean 17 and standard deviation 1

E. Mean 17 and standard deviation 5

3. A summer resort rents rowboats to customers but does not allow more than four people to a boat. Each boat is designed to hold no more than 800 pounds. Suppose the distribution of adult males who rent boats, including their clothes and gear, is normal with a mean of 190 pounds and standard deviation of 10 pounds. If the weights of individual passengers are independent, what is the probability that a group of four adult male passengers will exceed the acceptable weight limit of 800 pounds?

   A. 0.023

   B. 0.046

   C. 0.159

   D. 0.317

   E. 0.977

4. Carly commutes to work, and her commute time is dependent on the weather. When the weather is good, the distribution of her commute times is approximately normal with mean 20 minutes and standard deviation 2 minutes. When the weather is not good, the distribution of her commute times is approximately normal with mean 30 minutes and standard deviation 4 minutes. Suppose the probability that the weather will be good tomorrow is 0.9. Which of the following is closest to the probability that Carly's commute time tomorrow will be greater than 25 minutes?

   A. 0.0056

   B. 0.0894

   C. 0.0950

   D. 0.8055

   E. 0.9006

5. Sean and Evan are college roommates who have part-time jobs as servers in restaurants. The distribution of Sean's weekly income is approximately normal with mean \$225 and standard deviation \$25. The distribution of Evan's weekly income is approximately normal with mean \$240 and standard deviation \$15. Assuming their weekly incomes are independent of each other, which of the following is closest to the probability that Sean will have a greater income than Evan in a randomly selected week?

    A. 0.067

    B. 0.159

    C. 0.227

    D. 0.303

    E. 0.354

6. A company ships gift baskets that contain apples and pears. The distributions of weight for the apples, the pears, and the baskets are each approximately normal. The mean and standard deviation for each distribution is shown in the table below. The weights of the items are assumed to be independent.

Item	Mean	Standard Deviation
Apple	4.72 ounces	0.20 ounce
Pear	5.41 ounces	0.18 ounce
Basket	13.25 ounces	1.88 ounces

Let the random variable W represent the total weight of 4 apples, 6 pears, and 1 basket. Which of the following is closest to the standard deviation of W?

A. 1.90 ounces

B. 1.97 ounces

C. 2.26 ounces

D. 3.76 ounces

E. 3.83 ounces

7. The Attila Barbell Company makes bars for weight lifting. The weights of the bars are independent and are normally distributed with a mean of 720 ounces (45 pounds) and a standard deviation of 4 ounces. The bars are shipped 10 in a box to the retailers. The weights of the empty boxes are normally distributed with a mean of 320 ounces and a standard deviation of 8 ounces. The weights of the boxes filled with 10 bars are expected to be normally distributed with a mean of 7,520 ounces and a standard deviation of

   A. $\sqrt{12}$ ounces

   B. $\sqrt{80}$ ounces

   C. $\sqrt{224}$ ounces

   D. 48 ounces

   E. $\sqrt{1,664}$ ounces

8. The continuous random variable N has a normal distribution with mean 7.5 and standard deviation 2.5. For which of the following is the probability equal to 0?

   A. $P(N = 8)$

   B. $P(N > 8)$

   C. $P(N < 8)$

   D. $P(7 < N < 8)$

   E. $P(N < 7)$ or $P(N > 8)$

4 Binomial and Geometric Distributions

1. A large store has a customer service department where customers can go to ask for help with store-related issues. According to store records, approximately 1/4 of all customers who go to the service department ask for help finding an item. Assume the reason each customer goes to the service department is independent from customer to customer. Based on the approximation, what is the probability that at least 1 of the next 4 customers who go to the service department will ask for help finding an item?

   A. $4\left(\frac{1}{4}\right)$

   B. $1-\left(\frac{1}{4}\right)^4$

   C. $1-\left(\frac{3}{4}\right)^4$

   D. $4\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^3$

   E. $\left(\frac{4}{4}\right)\left(\frac{3}{4}\right)\left(\frac{2}{4}\right)\left(\frac{1}{4}\right)$

2. Based on his past record, Luke, an archer for a college archery team, has a probability of 0.90 of hitting the inner ring of the target with a shot of the arrow. Assume that in one practice Luke will attempt 5 shots of the arrow and that each shot is independent from the others. Let the random variable X represent the number of times he hits the inner ring of the target in 5 attempts. What is the probability that the number of times Luke will hit the inner ring of the target out of the 5 attempts is less than the mean of X?

   A. 0.40951

   B. 0.50000

   C. 0.59049

   D. 0.91854

   E. 0.99144

3. Mateo plays on his school basketball team. From past history, he knows that his probability of making a basket on a free throw is 0.8. Suppose he wants to create a simulation using random numbers to estimate the probability of making at least 3 baskets on his next 5 free throw attempts. Which of the following assignments of the digits 0 to 9 could be used for the simulation?

   A. Let the even digits represent making a basket and the odd digits represent not making a basket.

   B. Let the digits 0 and 1 represent making a basket and the digits from 2 to 9 represent not making a basket.

   C. Let the digits from 0 to 3 represent making a basket and the digits from 4 to 9 represent not making a basket.

   D. Let the digits from 0 to 6 represent making a basket and the digits from 7 to 9 represent not making a basket.

   E. Let the digits from 0 to 7 represent making a basket and the digits 8 and 9 represent not making a basket.

4. According to a recent survey, 31 percent of the residents of a certain state who are age 25 years or older have a bachelor's degree. A random sample of 50 residents of the state, age 25 years or older, will be selected. Let the random variable B represent the number in the sample who have a bachelor's degree. What is the probability that B will equal 40?

A. $\binom{50}{40}(0.31)^{40}(0.69)^{10}$

B. $\binom{50}{40}(0.69)^{40}(0.31)^{10}$

C. $\binom{40}{10}(0.31)^{40}(0.69)^{10}$

D. $\binom{40}{10}(0.69)^{40}(0.31)^{10}$

E. $40(0.31)^{50}$

5. A blind taste test will be conducted with 9 volunteers to determine whether people can taste a difference between bottled water and tap water. Each participant will taste the water from two different glasses and then identify which glass he or she thinks contains the tap water. Assuming that people cannot taste a difference between bottled water and tap water, what is the probability that at least 8 of the 9 participants will correctly identify the tap water?

   A. 0.0020

   B. 0.0195

   C. 0.8889

   D. 0.9805

   E. 0.9980

6. In a certain board game, a player rolls two fair six-sided dice until the player rolls doubles (where the value on each die is the same). The probability of rolling doubles with one roll of two fair six-sided dice is $\frac{1}{6}$. What is the probability that it takes three rolls until the player rolls doubles?

A. $\left(\frac{1}{6}\right)^3$

B. $\left(\frac{5}{6}\right)^3$

C. $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3$

D. $\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2$

E. $\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^2$

7. A company that ships crystal bowls claims that bowls arrive undamaged in 95 percent of the shipments. Let the random variable G represent the number of shipments with undamaged bowls in 25 randomly selected shipments. Random variable G follows a binomial distribution with a mean of 23.75 shipments and a standard deviation of approximately 1.09 shipments. Which of the following is the best interpretation of the mean?

   A. Every shipment of 25 bowls will have 23.75 undamaged bowls.

   B. Every shipment of 25 bowls will have 23.75 damaged bowls.

   C. On average, the company receives 23.75 shipments before receiving the first shipment with a damaged bowl.

   D. For all possible shipments of size 25, the average number of damaged shipments is equal to 23.75.

   E. For all possible shipments of size 25, the average number of undamaged shipments is equal to 23.75.

8. The transaction history at an electronic goods store indicates that 21 percent of customers purchase the extended warranty when they buy an eligible item. Suppose customers who buy eligible items are chosen at random, one at a time, until one is found who purchased the extended warranty. Let the random variable X represent the number of customers it takes to find one who purchased the extended warranty. Assume customers' decisions on whether to purchase the extended warranty are independent. Which of the following is closest to the probability that $X > 3$?

A. 0.131

B. 0.390

C. 0.493

D. 0.507

E. 0.624

9. According to a recent survey, 81 percent of adults in a certain state have graduated from high school. If 15 adults from the state are selected at random, what is the probability that 5 of them have not graduated from high school?

A. $\binom{20}{15}(0.19)^{15}(0.81)^{5}$

B. $\binom{10}{5}(0.19)^{15}(0.81)^{15}$

C. $\binom{10}{5}(0.81)^{5}(0.19)^{10}$

D. $\binom{15}{5}(0.19)^{5}(0.81)^{10}$

E. $\binom{15}{5}(0.81)^{5}(0.19)^{10}$

10. A recent study reported that 45 percent of adults in the United States now get all their news online. A random sample of 8 adults in the United States will be selected. What is the probability that 6 of the selected adults get all their news online?

A. $\binom{6}{2}(0.45)^8(0.55)^2$

B. $\binom{6}{2}(0.45)^6(0.55)^2$

C. $\binom{8}{6}(0.45)^2(0.55)^8$

D. $\binom{8}{6}(0.45)^6(0.55)^2$

E. $\binom{8}{6}(0.45)^8(0.55)^6$

5 Sampling Distributions

1. For which of the following conditions is it not appropriate to assume that the sampling distribution of the sample mean is approximately normal?

    A. A random sample of 8 taken from a normally distributed population

    B. A random sample of 50 taken from a normally distributed population

    C. A random sample of 10 taken from a population distribution that is skewed to the right

    D. A random sample of 75 taken from a population distribution that is skewed to the left

    E. A random sample of 100 taken from a population that is uniform

2. Researchers working for a certain airline are investigating the weight of carry-on bags. The researchers will use the mean weight of a random sample of 800 carry-on bags to estimate the mean weight of all carry-on bags for the airline. Which of the following best describes the effect on the bias and the variance of the estimator if the researchers increase the sample size to 1,300?

A. The bias will decrease and the variance will remain the same.

B. The bias will increase and the variance will remain the same.

C. The bias will remain the same and the variance will decrease.

D. The bias will remain the same and the variance will increase.

E. The bias will decrease and the variance will decrease.

3. A bag contains chips of which 27.5 percent are blue. A random sample of 5 chips will be selected one at a time and with replacement. What are the mean and standard deviation of the sampling distribution of the sample proportion of blue chips for samples of size 5?

A. The mean is 5(0.275), and the standard deviation is $\sqrt{5(0.275)(0.725)}$.

B. The mean is 0.275, and the standard deviation is $\sqrt{5(0.275)(0.725)}$.

C. The mean is 0.275, and the standard deviation is $\sqrt{\frac{0.275(0.725)}{5}}$.

D. The mean is 27.5, and the standard deviation is $\sqrt{5(27.5)(72.5)}$.

E. The mean is 27.5, and the standard deviation is $\sqrt{\frac{27.5(72.5)}{5}}$.

4. In two common species of flowers, A and B, the proportions of flowers that are blue are p_a and p_b, respectively. Suppose that independent random samples of 50 species-A flowers and 100 species-B flowers are selected. Let $\hat{p}_a$ be the sample proportion of blue species-A flowers and $\hat{p}_b$ be the sample proportion of blue species-B flowers. What is the mean of the sampling distribution of $\hat{p}_a - \hat{p}_b$?

   A. $p_a - p_b$

   B. $\frac{p_a}{50} - \frac{p_b}{100}$

   C. $\hat{p}_a - \hat{p}_b$

   D. $\frac{p_a(1-p_a)}{50} + \frac{p_b(1-p_b)}{100}$

   E. $\sqrt{\frac{p_a(1-p_a)}{50} + \frac{p_b(1-p_b)}{100}}$

5. Suppose the variance in trunk diameter of the giant sequoia tree species is 15.7 m^2, while the variance in trunk diameter of the California redwood tree species is 10.6 m^2. Let $\bar{x}_1$ represent the average trunk diameter of four randomly sampled giant sequoia trees, and let $\bar{x}_2$ represent the average trunk diameter of three randomly sampled California redwood trees. If the random sampling is done with replacement, what is the standard deviation $\sigma_{(\bar{x}_1 - \bar{x}_2)}$ of the sampling distribution of the difference in sample means $\bar{x}_1 - \bar{x}_2$?

A. $\sqrt{\frac{15.7}{4} - \frac{10.6}{3}}$

B. $\sqrt{15.7 - 10.6}$

C. $\sqrt{\frac{15.7}{4} + \frac{10.6}{3}}$

D. $\sqrt{\frac{15.7}{4}} + \sqrt{\frac{10.6}{3}}$

E. $\frac{15.7}{4} + \frac{10.6}{3}$

6. A manufacturer of cell phone batteries claims that the average number of recharge cycles for its batteries is 400. A consumer group will obtain a random sample of 100 of the manufacturer's batteries and will calculate the mean number of recharge cycles. Which of the following statements is justified by the central limit theorem?

   A. The distribution of the number of recharge cycles for the sample is approximately normal because the population mean of 400 is greater than 30.

   B. The distribution of the number of recharge cycles for the sample is approximately normal because the sample size of 100 is greater than 30.

   C. The distribution of the number of recharge cycles for the population is approximately normal because the sample size of 100 is greater than 30.

   D. The distribution of the sample means of the number of recharge cycles is approximately normal because the sample size of 100 is greater than 30.

   E. The distribution of the sample means of the number of recharge cycles is approximately normal because the population mean of 400 is greater than 30.

7. The normal curve shown represents the sampling distribution of a sample mean for sample size $n = 25$, selected at random from a population with standard deviation σ_x. (Refer to the image in the original prompt where a normal curve is shown with an x-axis span.) Which of the following is the best estimate of the standard deviation of the population, σ_x?

A. 3

B. 6

C. 15

D. 30

E. 75

8. Based on records kept at a gas station, the distribution of gallons of gas purchased by customers is skewed to the right with mean 10 gallons and standard deviation 4 gallons. A random sample of 64 customer receipts was selected, and the sample mean number of gallons was recorded. Suppose the process of selecting a random sample of 64 receipts and recording the sample mean number of gallons was repeated for a total of 100 samples. Which of the following is the best description of a dotplot created from the 100 sample means?

   A. The dotplot is skewed to the right with mean 10 gallons and standard deviation 4 gallons.

   B. The dotplot is skewed to the right with mean 10 gallons and standard deviation 0.5 gallon.

   C. The dotplot is skewed to the right with mean 10 gallons and standard deviation 0.4 gallon.

   D. The dotplot is approximately normal with mean 10 gallons and standard deviation 0.5 gallon.

   E. The dotplot is approximately normal with mean 10 gallons and standard deviation 0.4 gallon.

9. The director of a marketing department wants to estimate the proportion of people who purchase a certain product online. The director originally planned to obtain a random sample of 2,500 people who purchased the product. However, because of budget concerns, the sample size will be reduced to 1,500 people. Which of the following describes the effect of reducing the number of people in the sample?

   A. The variance of the sample will increase.
   B. The variance of the population will decrease.
   C. The variance of the sampling distribution of the estimator will increase.
   D. The variance of the sampling distribution of the estimator will decrease.
   E. The variance of the sampling distribution of the estimator will remain the same.

10. The graph shows the population distribution of random variable X with mean 85 and standard deviation 18. (The graph shows a skewed distribution). Which of the following graphs is a sampling distribution of the sample mean $\bar{x}$ for samples of size 40 taken from the population?

A. A graph similar to the population (skewed).

B. A graph slightly less skewed.

C. An approximately normal distribution centered at 85 with smaller spread.

D. An approximately normal distribution centered at 85 with larger spread.

E. A uniform distribution.

(Note: Based on CLT, for $n = 40$, the sampling distribution should be approximately normal).

11. Two different drugs, X and Y, are currently in use to treat a certain condition. About 7 percent of the people using either drug experience side effects. A random sample of 75 people using drug X and a random sample of 150 people using drug Y are selected. The proportion of people in each sample who experience side effects is recorded. Are the sample sizes large enough to assume that the sampling distribution of the difference in sample proportions is approximately normal?

A. Yes. Both sample sizes are large enough.

B. No. The sample size for drug X is large enough, but the sample size for drug Y is not.

C. No. The sample size for drug Y is large enough, but the sample size for drug X is not.

D. No. Neither sample size is large enough.

E. There is not enough information provided to determine whether the sampling distribution is approximately normal.

12. A recent survey concluded that the proportion of American teenagers who have a cell phone is 0.27. The true population proportion of American teenagers who have a cell phone is 0.29. For samples of size 1,000 that are selected at random from this population, what are the mean and standard deviation, respectively, for the sampling distribution of the sample proportion of American teenagers who have a cell phone?

A. 0.27, $\sqrt{1000(0.27)(0.73)}$

B. 0.27, $\sqrt{\frac{(0.29)(0.71)}{1000}}$

C. 0.27, $\sqrt{\frac{(0.27)(0.73)}{1000}}$

D. 0.29, $\sqrt{\frac{(0.29)(0.71)}{1000}}$

E. 0.29, $\sqrt{1000(0.29)(0.71)}$

13. A reading specialist wanted to estimate the mean word length, in number of letters, for an elementary school history textbook. The specialist took repeated random samples of size 100 words and estimated the mean and standard deviation of the sampling distribution to be 4.9 letters and 0.15 letter, respectively. Based on the estimates for the sampling distribution, which of the following provides the best estimates of the population parameters?

   A. Mean 4.9 letters and standard deviation 0.015 letter

   B. Mean 4.9 letters and standard deviation 0.15 letter

   C. Mean 4.9 letters and standard deviation 1.5 letters

   D. Mean 0.49 letter and standard deviation 0.15 letter

   E. Mean 49 letters and standard deviation 15 letters